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Hartle-Hawking vacuum for $c = 1$ tachyon condensation

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Abstract: Quantum vacua are constructed for a time-symmetric cosmology describing closed string tachyon condensation in two-dimensional string theory. Due to the euclidean periodicity of the solution, and despite its time dependence, we are able to construct thermal states at discrete values of the temperature. The asymptotic thermal Green functions and stress energy tensor are computed and found to have an intriguing resemblance to those in the Hartle-Hawking vacuum of a black hole.

Keywords: Tachyon Condensation, 2D Gravity, Matrix Models.

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1. Introduction

Soluble string theories in two dimensions provide a useful laboratory for studying time dependent phenomena such as closed or open string tachyon condensation and particle production [1]–[10]. Recently, studies appeared of particle production during closed string tachyon condensation [8, 9]. The “Alexandrov vacuum” was defined in which a tachyon wall distorts the quantum vacuum of the collective field (describing fluctuations of the Fermi surface) like a reflecting mirror. It was found that when the tachyon condenses, the mirror accelerates up to $\mathcal{I}^+$ and particles are produced.

In this paper we consider a closely related time-symmetric cosmology in which the tachyon is condensed in both the far future and the far past. The Alexandrov vacuum is constructed. Thinking of the tachyon wall as an analog of the black hole horizon, the Alexandrov vacuum corresponds to the Boulware vacuum. There is no energy flux across the tachyon wall (horizon). Observers comoving with the wall (horizon) will detect no particles. However, there is a negative Casimir energy for the collective field which diverges at the wall (horizon).

Further, we define a second, thermal, vacuum state. Thermal equilibrium is possible at a spectrum of discrete temperatures despite the time dependence, as in [11], because of the imaginary-time periodicity of the solution. The thermal energy of the collective field has a positive energy density which diverges at the tachyon wall due to an infinite blueshift. At a critical value of the temperature, this divergence is found to exactly cancel the negative-energy divergence in the Casimir energy, and the total stress energy of the collective field becomes finite. A mathematically identical cancellation was found by the authors of [9], who include a c-number correction to the collective field hamiltonian which is (apparently) not a thermal energy of the collective field. It would be interesting to better understand the relation of their observations to ours.

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A similar phenomenon occurs for the thermal Hartle-Hawking vacuum for Schwarzchild black holes. This vacuum can be represented as a thermal ensemble of Boulware particles built on the Boulware vacuum. In this description the negative energy Casimir divergence at the horizon is cancelled by the infinitely blueshifted energy density of the thermal bath. Although the analogy is not complete, we accordingly refer to the thermal matrix model vacuum at the critical temperature as the Hartle-Hawking vacuum.

It seems self evident that these observations are somehow relevant to the exasperating problem of understanding the role of black holes in the matrix model (see e.g. [12]–[16]). However, we have nothing concrete to offer in this direction, and accordingly leave this problem to the avid reader.

In the next section we adapt the discussion of [8, 9], constructing the Alexandrov vacuum of the collective field for our time-symmetric cosmology. In section 3 we construct the thermal state in both the collective field and exact free fermion pictures. Asymptotic correlators are computed. Thermality requires that both sides of the fermion barrier are populated, which in turn entails the 0B interpretation of the matrix model [17, 18]. Familiarity with the free fermion and Das-Jevicki collective field formalisms is assumed throughout (a useful recent reference is [19]), and our notation is exactly that of [8].

2. The Alexandrov/Boulware vacuum

We are interested in the following solution to the equations of motion for the Fermi surface

$$(x + p - 2\lambda e^{t})(x - p - 2\lambda e^{-t}) = 2\mu\,, \tag{2.1}$$

where x and p parameterize the phase space of the free fermions, and μ, λ are non-negative constants. This solution describes a time-reversal invariant cosmological solution in which the tachyon is condensed on a future portion of $\mathcal{I}^+$ and a past portion of $\mathcal{I}^-$. To see this, note that the Fermi surface in this case is a moving hyperbola centered at

$$(x, p) = (2\lambda \cosh t, 2\lambda \sinh t)\,. \tag{2.2}$$

Thus the right branch of the Fermi sea is filled at time $t = 0$, but drains in the far past and far future, as the hyperbola moves out to large positive x. This differs from the solution considered in [8], which had tachyon condensation in the future but not in the past.

In this section we will use the Das-Jevicki formalism to study collective fluctuations of this Fermi surface, and describe the vacuum states for the system. In particular, we will construct the "Alexandrov vacuum" for (2.1), following the discussion of [8].

The hyperbola (2.1) has a second branch centered at the same point (2.2) as the first, describing the Fermi surface spilling over from the left hand side of the barrier. In this section we will consider the bosonic string and ignore the left side of the Fermi sea (which is consistent in perturbation theory), although it may be easily analyzed. However, the thermal states considered in the next section necessarily thermally populate both sides of the barrier, so the left branch of the Fermi sea must be considered. This will force us to the type 0B interpretation.

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2.1 Fluctuations of the Fermi surface

We wish to describe collective fluctuations of the Fermi field in Das-Jevicki formalism [20], following the conventions and notation of [8]. We may parameterize the right hand branch of the solution to (2.1) by introducing a coordinate σ that runs from $-\infty$ to ∞, and taking

$$\begin{aligned} x &= \sqrt{2\mu}\cosh\sigma + 2\lambda\cosh t \\ p &= \sqrt{2\mu}\sinh\sigma + 2\lambda\sinh t\,. \end{aligned} \tag{2.3}$$

Introducing the collective field φ, which is the difference between the upper and lower Fermi surfaces

$$\varphi = \frac{1}{2}(p_+ - p_-) \tag{2.4}$$

we find that the background solution (2.1) corresponds to

$$\varphi_0 = \sqrt{(x-2\lambda\cosh t)^2 - 2\mu}\,. \tag{2.5}$$

To describe small fluctuations around this background we define the field η by

$$\varphi = \varphi_0 + \sqrt{\pi}\partial_x\eta\,. \tag{2.6}$$

The dynamics of the small fluctuations are governed by the action [20]

$$S = \int dtdx\left[\frac{1}{2\pi}\frac{(Z_0+\sqrt{\pi}\partial_t\eta)^2}{\varphi_0+\sqrt{\pi}\partial_x\eta} - \frac{1}{6\pi}(\varphi_0+\sqrt{\pi}\partial_x\eta)^3 + \frac{1}{\pi}\left(\frac{1}{2}x^2-\mu\right)(\varphi_0+\sqrt{\pi}\partial_x\eta)\right], \tag{2.7}$$

where

$$Z_0 = \int^x dx'\partial_t\phi_0(x') = -2\lambda\sinh t\;\varphi_0\,. \tag{2.8}$$

The quadratic part of this action

$$\begin{aligned} S_2 &= \frac{1}{2}\int\frac{dxdt}{\sqrt{(x-2\lambda\cosh t)^2-2\mu}}\times \\ &\quad\times\left[(\partial_t\eta)^2 + 4\lambda\sinh t(\partial_t\eta\partial_x\eta) - (x^2-4x\lambda\cosh t - 2\mu + 4\lambda^2)(\partial_x\eta)^2\right] \end{aligned} \tag{2.9}$$

is that of a scalar field η in a curved two dimensional space. Note from (2.3) that the Fermi sea extends from $\sqrt{2\mu}+2\lambda\cosh t < x < \infty$, so it is natural to impose reflecting boundary conditions on the field η along the "mirror" trajectory

$$x = \sqrt{2\mu} + 2\lambda\cosh t\,. \tag{2.10}$$

The action simplifies considerably when written in terms of the Alexandrov coordinates (τ,σ) or $\tau^\pm = \tau\pm\sigma$, defined by [19]

$$t=\tau\,,\qquad x = \sqrt{2\mu}\cosh\sigma + 2\lambda\cosh\tau\,. \tag{2.11}$$

The full action (2.7) becomes

$$\begin{aligned} &\int d\tau d\sigma\left[\frac{1}{2}((\partial_\tau\eta)^2-(\partial_\sigma\eta)^2) - \frac{\sqrt{\pi}}{6\varphi_0^2}(3(\partial_\tau\eta)^2(\partial_\sigma\eta)+(\partial_\sigma\eta)^3)\right. \\ &\qquad\left. + \sum_{n=2}^{\infty}\frac{(-1)^n}{2}(\partial_\tau\eta)^2\left(\frac{\sqrt{\pi}(\partial_\sigma\eta)}{\varphi_0^2}\right)^n\right], \end{aligned} \tag{2.12}$$

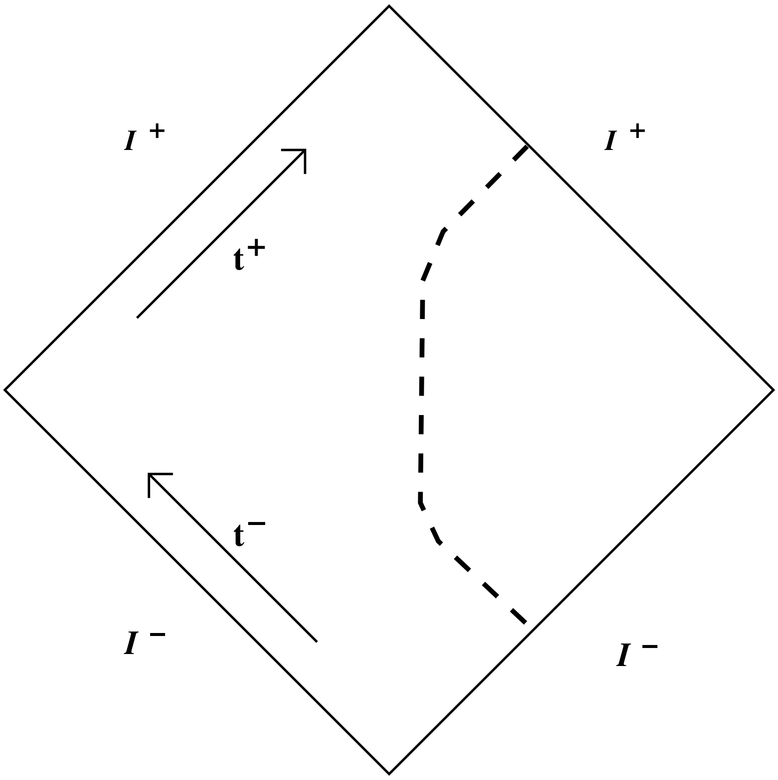

Figure 1: Penrose diagram for spacetime creation/decay via tachyon condensation. The dashed line represents the tachyon wall, which asymptotes to $t^+ = \ln \lambda$ on $\mathcal{I}^-$ and $t^- = -\ln \lambda$ on $\mathcal{I}^+$.

with $\varphi_0 = \sqrt{2\mu} \sinh \sigma$. Note that the quadratic part of the action is precisely that of a scalar field η in flat space $-d\tau^2 + d\sigma^2$. Moreover, the mirror trajectory (2.10) is just the line $\sigma = 0$.

We will also consider the "fermion" coordinates

$$t^{\pm} = t \pm q\,, \qquad q = \ln x \tag{2.13}$$

which are relevant for the spacetime description of this process near the asymptotic boundaries. The transformation to the Alexandrov coordinates becomes especially simple on $\mathcal{I}^{\pm}$; on $\mathcal{I}^+$, as $t^+ \to \infty$,

$$\tau^+ \to \infty\,, \qquad \tau^- = -\ln\left(\sqrt{\frac{2}{\mu}}(e^{-t^-} - \lambda)\right). \tag{2.14}$$

On $\mathcal{I}^+$, the mirror is located at the point $t^- = -\ln \lambda$, where the coordinate transformation (2.14) becomes singular.

2.2 Vacuum states

We will now describe vacuum states of the collective field theory defined in the previous section. For the rest of this section we will discuss the free η field theory, ignoring the higher order interactions appearing in (2.7) and (2.12). The results of this simplified analysis will turn out to agree with the full Fermi picture, as we will see in the next section.

There are two natural vacuum states $|in\rangle$ and $|out\rangle$, associated to the positive frequency modes on $\mathcal{I}^{\pm}$

$$
\begin{aligned}
u_\omega^{in} &\to \frac{1}{\sqrt{2\omega}} e^{-i\omega t^+}\,, \qquad t^- \to -\infty \\
u_\omega^{out} &\to \frac{1}{\sqrt{2\omega}} e^{-i\omega t^-}\,, \qquad t^+ \to \infty\,.
\end{aligned}
\tag{2.15}
$$

These states have no particles in the far past and far future, respectively. They are not equal, but are related by time reversal $t \to -t$.

Following [8], we may also define the Alexandrov vacuum $|0_A\rangle$ associated to the Alexandrov coordinate plane waves

$$
u_\omega^0 = \frac{1}{2\omega}(e^{-i\omega\tau^+} + e^{-i\omega\tau^-})\,. \tag{2.16}
$$

This state is time reversal invariant, and differs from $|in\rangle$ and $|out\rangle$. For example, on $\mathcal{I}^+$ the outgoing Alexandrov modes depend on t^- as

$$
u_\omega^0 \to \left(\frac{e^{t^-} - \lambda}{\sqrt{2\omega}}\right)^{i\omega}\,. \tag{2.17}
$$

The incoming Alexandrov modes take a similar form on $\mathcal{I}^-$. We conclude that $|0_A\rangle$ is an excited squeezed state on both $\mathcal{I}^{\pm}$.

The energy flux on $\mathcal{I}^+$ in the Alexandrov state is given by the schwarzian of the transformation (2.14) between Alexandrov and t^- coordinates on $\mathcal{I}^+$,

$$
\begin{aligned}
T_{--}(t^-) &= -\frac{1}{12}(\frac{\partial\tau^-}{\partial t^-})^{3/2}\frac{\partial^2}{\partial\tau^{-2}}(\frac{\partial\tau^-}{\partial t^-})^{1/2} \\
&= -\frac{\lambda e^{t^-}(2-\lambda e^{t^-})}{48(1-\lambda e^{t^-})^2}\,.
\end{aligned}
\tag{2.18}
$$

Note that this diverges at the mirror $t^- = -\ln\lambda$. This is the same as the result in [8]. However, unlike [8], a similar nontrivial schwarzian gives an energy flux on $\mathcal{I}^-$, which diverges at the mirror as well.

3. Thermal states

In this section we will describe thermal states of the field theory defined in section 2. In a thermal ensemble both sides of the barrier will be filled and one must include the second branch of the hyperbola, suppressed in the previous section. While the right branch describes a draining of the right Fermi sea, the left branch describes the Fermi sea spilling over the barrier at late and early times. The region $t^- > -\ln\lambda$, left behind by the draining Fermi sea, is filled by the spilling sea, and all regions of $\mathcal{I}$ have a Fermi surface whose fluctuations are the collective field η.

Once we consider both sides of the Fermi sea, we are forced to the type 0B interpretation [17, 18] of the matrix model. In this interpretation, left-right symmetric η fluctuations correspond (after a leg pole transformation) to the NS-NS scalar, while antisymmetric

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fluctuations correspond to the RR scalar. In this paper we will present the correlators in their simplest form, namely in terms of the collective fields for the left and right Fermi surfaces.

Formulae for the 0B leg pole transforms can be found in [17, 18]. The leg pole transformation to spacetime scalars differs for the symmetric and antisymmetric modes. These leg pole transformations act as the identity on very long wavelength fluctuations but smear correlators on the string scale. In practice they are not analytically computable for the correlators given here. Additionally, it is not clear how the standard leg-pole transformations are to be adapted to non-perturbatively non-trivial situations of the type discussed herein. One possibility was explored in [8].

3.1 Stress energy tensor

The tachyon wall trajectory

$$x = \sqrt{2\mu} + 2\lambda \cosh t \tag{3.1}$$

is invariant under shifts in imaginary time

$$t \to t + 2\pi i n \tag{3.2}$$

for any integer n. This means [11] that we can define a discrete set of thermal states with temperatures

$$T = \frac{1}{2\pi n}\,, \tag{3.3}$$

as we will see in the next subsection. From (2.11) we can see that a periodic identification of imaginary time t leads to a periodic identification of imaginary τ. Thus in Alexandrov coordinates these thermal states are defined by the usual periodicity

$$\tau = \tau + 2\pi i n\,. \tag{3.4}$$

This periodic identification in euclidean space leads to a thermal energy density

$$T_{--} = \frac{1}{48n^2} \tag{3.5}$$

in the τ^- coordinates. The schwarzian to t^- coordinates gives the energy on $\mathcal{I}^+$

$$\begin{aligned} T(t^-) &= \frac{1}{48n^2}\left(\frac{d\tau^-}{dt^-}\right)^2 - \frac{1}{12}(\frac{\partial\tau^-}{\partial t^-})^{3/2}\frac{\partial^2}{\partial\tau^{-2}}(\frac{\partial\tau^-}{\partial t^-})^{1/2} \\ &= \frac{1}{48} + \frac{n^{-2}-1}{48(1-\lambda e^{t^-})^2}\,. \end{aligned} \tag{3.6}$$

This is a constant for $n = 1$, indicating that at temperature $T = 1/2\pi$ the stress tensor is finite as one approaches the mirror — in particular, it is constant everywhere on $\mathcal{I}^+$. In sections 3.2 and 3.3 we will study this thermal behavior in more detail by calculating two point functions on $\mathcal{I}$. Finally, in section 3.4 we will discuss the full free fermion description of these thermal states, and use this formalism to describe this thermal state in more detail.

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Since we are studying here the dynamics and quantum state of the collective field, we do not include in our expression for collective field stress energy the counterterm corresponding to constant one loop energy density as derived in [21]. The self-consistency of this approach is confirmed by the fact that constant stress energy density at $n = 1$ corresponds to thermal two point functions, as will be seen.

3.2 Green functions

We will now study the Green functions of the pure and thermal states in the collective field formalism.

First, consider the Alexandrov vacuum $|0_A\rangle$. In the τ coordinates the quadratic part of the action (2.12) is just that of a scalar field in flat space with a mirror located at $\sigma = 0$. The two point function is[1]

$$\begin{aligned} G(\tau,\sigma;\tau',\sigma') &= \langle 0|\partial_{\tau^-}\eta(\tau,\sigma)\partial_{\tau^{-\prime}}\eta(\tau',\sigma')|0\rangle \\ &= \frac{1}{4\pi}\left(\frac{1}{(\tau-\tau'-\sigma+\sigma')^2}+\frac{1}{(\tau-\tau'-\sigma-\sigma')^2}\right). \end{aligned} \tag{3.7}$$

The first term is the second derivative of the usual two point function of a boson in two dimensions. The second comes from the boundary conditions at the mirror — it corresponds to the image charge located at $-\sigma'$.

Thermal Green functions are found by imposing periodicity in imaginary time. So we may use the standard relation between pure state and thermal propagators

$$G_\beta(\tau,\sigma;\tau',\sigma') = \sum_m G(\tau,\sigma;\tau'+mi\beta,\sigma'). \tag{3.8}$$

This leads to the thermal propagator

$$G_\beta(\tau,\sigma;\tau',\sigma') = \frac{\pi}{4\beta^2}\left(\frac{1}{\sinh^2\pi(\tau-\tau'-\sigma+\sigma')/\beta}+\frac{1}{\sinh^2\pi(\tau-\tau'-\sigma-\sigma')/\beta}\right). \tag{3.9}$$

We can now transform this back to the (t,x) coordinates. For $\beta = 2\pi n$ we can perform the sum (3.8) in either the $\tau^\pm$ or $t^\pm$ coordinates, since the coordinate σ defined by (2.11) is invariant under the shift $t \to t+i\beta$. The thermal propagator becomes

$$\begin{aligned} G_\beta(t,x;t',x') &= \langle 0|\partial_{t^-}\eta\partial_{t^{-\prime}}\eta|0\rangle \\ &= JJ'\frac{\pi}{4\beta^2}\left(\frac{1}{\sinh^2\pi(t-t'-\sigma+\sigma')/\beta}+\frac{1}{\sinh^2\pi(t-t'-\sigma-\sigma')/\beta}\right), \end{aligned} \tag{3.10}$$

where

$$\sigma = \cosh^{-1}\frac{x-2\lambda\cosh t}{\sqrt{2\mu}}. \tag{3.11}$$

We are considering here the two point function of $\partial_{t^-}\eta$ rather than $\partial_{\tau^-}\eta$, so we must include the jacobian J evaluated at constant t^+

$$J = \left(\frac{\partial\tau^-}{\partial t^-}\right)^{-1}_{t^+}. \tag{3.12}$$

[1] We consider here correlators of the derivative of η, rather than just η. This two point function has dimension two, so is the natural object to study in two dimensions.

This Green function takes a particular simple form on $\mathcal{I}^+$. First note that σ is only well defined in the region of $\mathcal{I}^+$ with

$$-\infty < t_- < -\ln\lambda\,, \tag{3.13}$$

where

$$\sigma \to \ln\sqrt{\frac{2}{\mu}} + t + \ln(e^{-t^-} - \lambda) \to \infty\,. \tag{3.14}$$

The second term in (3.10) then vanishes as e^{-t^+} , and the first term gives

$$G_\beta = JJ' \frac{\pi}{4\beta^2 \sinh^2 \pi \ln\left(\frac{e^{-t^{-\prime}}-\lambda}{e^{-t^-}-\lambda}\right)/\beta}\,. \tag{3.15}$$

When $n = 1$, so that $\beta = 2\pi$, several terms cancel and this becomes the usual thermal propagator in two dimensions

$$G_{\beta=2\pi} = \frac{1}{16\pi \sinh^2(t^- - t^{-\prime})/2}\,. \tag{3.16}$$

We conclude that the physical tachyon correlation functions on the boundary with both points in the region (3.13) are precisely thermal. The case where both points are not in this region is more interesting, as we will now discuss.

3.3 Correlators on all of $\mathcal{I}^+$

To study correlations on the rest of $\mathcal{I}^+$, it is useful to define a coordinate r which covers all of $\mathcal{I}^+$ — we will take

$$\begin{aligned} r &= -\exp(-\tau^+) \qquad \text{as} \qquad \tau^- \to +\infty\,, \\ r &= \exp(-\tau^-) \qquad \text{as} \qquad \tau^+ \to +\infty\,. \end{aligned} \tag{3.17}$$

We can define a similar coordinate R which is related to the fermion coordinates $t^\pm$,

$$\begin{aligned} R &= -\exp(-t^+) \qquad \text{as} \qquad t^- \to +\infty\,, \\ R &= \exp(-t^-) \qquad \text{as} \qquad t^+ \to +\infty\,. \end{aligned} \tag{3.18}$$

So $R \sim xe^{-t}$ as $t \to +\infty$. These coordinates are useful because the coordinate change from Alexandrov to fermion coordinates is particularly simple: on $\mathcal{I}^+$, $R = r + \lambda$. For $n = 1, \beta = 2\pi$, the thermal two point function on the left hand component of $\mathcal{I}^+$ (where $\tau^- \to \infty$) reduces to

$$G_{\beta=2\pi}(\tau^+, \tau'^+) \equiv \langle 0|\partial_{\tau^+}\eta(\tau^+)\partial_{\tau^{+\prime}}\eta(\tau'^+)|0\rangle = \frac{1}{16\pi}\frac{1}{\sinh^2\left((\tau^+ - \tau'^+)/2\right)}\,. \tag{3.19}$$

On the right hand part of $\mathcal{I}^+$ ($\tau^+ \to \infty$),

$$G_{\beta=2\pi}(\tau^-, \tau'^-) \equiv \langle 0|\partial_{\tau^-}\eta(\tau^-)\partial_{\tau^{-\prime}}\eta(\tau'^-)|0\rangle = \frac{1}{16\pi}\frac{1}{\sinh^2\left((\tau^- - \tau'^-)/2\right)}\,. \tag{3.20}$$

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The correlator when one point lies on the right hand component of $\mathcal{I}^+$ and the other lies on the left hand component vanishes,

$$G_{\beta=2\pi}(\tau^+, \tau'^-) \equiv \langle 0|\partial_{\tau^+}\eta(\tau^+)\partial_{\tau^-{}'}\eta(\tau'^-)|0\rangle = 0\,. \tag{3.21}$$

These three formulas are compactly written in the r coordinate as

$$G_{\beta=2\pi}(r, r') = \frac{1}{16\pi}\frac{(r-r')^2}{rr'}\,\Theta(rr')\,. \tag{3.22}$$

Transforming to the fermion R coordinate, using the jacobians $\partial r/\partial\tau^- = \partial r/\partial\tau^- = -r$ and $\partial R/\partial t^- = \partial R/\partial t^- = -R$, the correlator becomes

$$G_{\beta=2\pi}(R, R') = \frac{1}{16\pi}\frac{(R-R')^2}{RR'}\,\Theta((R-\lambda)(R'-\lambda))\,. \tag{3.23}$$

This reproduces the thermal correlator (3.16) when both points lie on the same component of $\mathcal{I}^+$, and allows one to compute correlators between points in different components of $\mathcal{I}^+$. For example, in the fermion coordinates the correlator between one point on the right hand side of $\mathcal{I}^+$ and another on the left hand side of $\mathcal{I}^+$ (with $t^{-\prime} \to \infty$) does not vanish. In particular, if the point on the right hand side of $\mathcal{I}^+$ lies in the region with $t^- > -\ln(\lambda)$ then

$$G_{\beta=2\pi}(t^- > -\ln(\lambda), t'^+) = -\frac{1}{16\pi}\frac{1}{\cosh^2((t^- - t'^+)/2)}\,. \tag{3.24}$$

These correlators are exactly thermal, but with a reflecting mirror inserted at $t^- = -\ln(\lambda)$ on the right hand side of $\mathcal{I}^+$. Correlators for the spacetime axion and tachyon of the 0B theory may be obtained from these via a leg pole transformation of the left-right antisymmetric and symmetric linear combinations of η, but we shall not give the explicit expressions.

3.4 Exact free fermion picture

In this subsection, we will show, using the free fermion description, that the thermal correlators derived above in the collective field formalism are actually exact. We will also give a precise meaning to the preceding statement that there exist thermal states with temperature $T = 1/2\pi n$.

The difference between the quantum state describing the original static Fermi sea and the state describing the filling and draining of the Fermi sea is too large to be described as a state in the Hilbert space of the original theory [22]. In the language of [22], it involves non-normalizable modes. Hence it is associated to a new hamiltonian rather than to a semiclassical quantum state in the theory governed by the old hamiltonian. This new, time-dependent hamiltonian can be written as

$$H_\lambda = \frac{1}{2}(p^2 - x^2) + \lambda e^t(x-p) + \lambda e^{-t}(x+p)\,. \tag{3.25}$$

Consider the new phase space coordinates

$$(y, p_y) \equiv (x - 2\lambda\cosh t, p - 2\lambda\sinh t)\,, \tag{3.26}$$

which obey the usual equations of motion $\dot{y} = p_y$, $\dot{p}_y = y$. In these coordinates the hamiltonian is $(p_y^2 - y^2)/2$ and the Fermi surface (2.1) is given by $(y^2 - p_y^2) = 2\mu$. We know the quantum theory corresponding to this classical limit — it is simply the theory of free fermions with potential $-y^2/2$, with all states up to μ below the top filled. Thus exact correlators at finite λ are related to the correlators in the usual $\lambda = 0$ theory (which can be computed as in [23]) by the coordinate shift (3.26). This procedure reproduces the results derived above in the collective field approximation. The argument is a straightforward adaptation of the one given in [8] and will not be repeated here.

We may also use the free fermion picture to understand the physical meaning of these thermal states. Typically, thermal states are only defined for time independent systems, where correlators calculated in a compact euclidean space $t \sim t + i\beta$ agree with those computed from a density matrix $\rho = e^{-\beta H}$. In our case, the hamiltonian (3.25) depends on time, so it is not clear which density matrix is associated to the euclidean identification $t \sim t + 2\pi i n$. In fact, as we will now show, the density matrix is

$$\rho = e^{-2\pi n H_0}\,, \tag{3.27}$$

where H_0 is the time independent part of the hamiltonian, and will be defined shortly.

To see this, we will use the second quantized language, where the fermion field $\Psi(x,t)$ can be expanded as

$$\Psi(x,t) = \sum_{\omega,s} a_\omega^{(s)} \psi_\omega^{(s)}(x - 2\lambda\cosh t, t)\,. \tag{3.28}$$

The wave functions $\psi_\omega^{(s)}(y,t)$, for $s = \pm 1$, are the symmetric/antisymmetric solutions to the one particle Schrodinger equation at energy ω

$$-\frac{1}{2}(\partial_y^2 + y^2)\psi_\omega^{(s)}(y,t) = -i\partial_t \psi_\omega^{(s)}(y,t) = \omega\psi_\omega^{(s)}(y,t)\,. \tag{3.29}$$

The second quantized hamiltonian

$$H(t) = \int dx \Psi^\dagger(x,t)\left[\frac{1}{2}(\partial_x^2 + x^2) + \lambda e^t(x + i\partial_x) + \lambda e^{-t}(x - i\partial_x) - 2\lambda^2\cosh^2 t\right]\Psi(x,t) \tag{3.30}$$

gives the Schrödinger evolution of Ψ

$$-i\partial_t\Psi(x,t) = [H(t), \Psi(x,t)]\,. \tag{3.31}$$

We define now the time independent portion of the hamiltonian, given by (up to a zero-point energy)

$$H_0 = \sum_{\omega,s} \left(a_\omega^{(s)}\right)^\dagger a_\omega^{(s)}\omega\,. \tag{3.32}$$

The operator H_0 generates the following shift in imaginary time

$$e^{2\pi n H_0}\Psi(x,t)e^{-2\pi n H_0} = \Psi(x, t + 2\pi i n) \tag{3.33}$$

since each wave function ψ_ω obeys

$$e^{-2\pi n\omega}\psi_\omega(x - 2\lambda\cosh t, t) = \psi_\omega(x - 2\lambda\cosh t', t')\Big|_{t' = t + 2\pi i n}\,. \tag{3.34}$$

Therefore, the Heisenberg-picture density matrix is given in equation (3.27). Correlation functions in this density matrix agree with those computed by analytic continuation from a periodically identified euclidean theory, as in [11].

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